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Neutron Stars:

As we discussed earlier, stars with an initial mass $8M_\odot \lesssim M \lesssim 25 M_\odot$ undergo core collapse after making ^{56}Fe, which results in type II supernova. The core will eventually stabilize in the form of a compact object called a "Neutron Star".

The simplest model for a neutron star is based on an ideal degenerate gas of non-relativistic neutrons, and assume Newtonian gravity. The equation of state is then given by an $n=\frac{3}{2}$ Polytrope, and we have the standard relation $M \propto R^{-3}$. Solving the Lane-Emden equation, we find:

$$\boxed{\begin{aligned} M &= 1.102\left(\frac{\rho_c}{10^{15}\,g\,cm^{-3}}\right)^{\frac{1}{2}} M_\odot = \left(\frac{15.12\,km}{R}\right)^3 M_\odot \\ R &= 14.64\left(\frac{\rho_c}{10^{15}\,g\,cm^{-3}}\right)^{-\frac{1}{6}} km \end{aligned}}$$

This is valid only when neutrons are stable against β-decay and contribute most of the pressure. This is possible only for $\rho \gtrsim \rho_{min}$, $\rho_{min} \simeq 4\times10^{12}\,g\,cm^{-3}$ where neutron drip gives rise to a large number of free neutrons. It is easy to show that at such densities, a small electron fraction will be sufficient to block β-decay according to the Pauli exclusion principle. Setting $\rho_c = \rho_{min}$, we then find:

$$M \simeq 0.1\,M_\odot \quad , \quad R \simeq 36\ \mathrm{km} \quad *$$

Recall that the maximum central density for a stable white dwarf is $\rho \simeq 10^{10}\,g\,cm^{-3}$. There is no stable compact object in the gap $10^{10}\,g\,cm^{-3} \lesssim \rho \lesssim 4\times10^{12}\,g\,cm^{-3}$. We also note that the minimum mass for a neutron star ($\simeq 0.1\,M_\odot$) is smaller than the maximum mass for a stable white dwarf M_{Ch}. Thus we can have completely different stable structures for the in-between mass range. For example, we can have a white dwarf or a neutron star with a mass of $0.4 M_\odot$, depending on the internal composition.

It is important to stress the following fact. Neutron stars are the most compact configurations of matter that can withstand gravitational force. They constitute the lowest possible energy state for an object with a certain number of baryons. This is because they arise as stellar remnants in which nuclear burning has produced ^{56}Fe nuclei in the core. On the other hand, white dwarfs are not the lowest possible energy configurations since nuclear reactions have not proceeded to the maximum limit there. However, they are quite stable because there is no possibility of further nuclear reactions taking place in the low-density (isolated) white dwarfs.

As a consequence, we can compute equation of state of neutron stars by taking into account the requirement of lowest energy (See pages 218 - 220). But this will not be the case for white dwarfs.

It is seen from equation * that neutron stars are very compact objects. This indicates that general relativistic effects cannot be neglected because of the strong gravitational fields that arise. This can be seen by calculating $\frac{GM}{Rc^2}$, which shows the quantity is not significantly small compared to unity. One therefore needs to use general relativity instead of Newtonian gravity to derive the M-R relation for a neutron star.

For a given equation of state, the M-R relation is determined by integrating the Tolman-Oppenheimer-Volkoff equation:

$$\frac{dP}{dr} = -\frac{G}{r^2}\left[\rho + \frac{P}{c^2}\right]\left[M_r + 4\pi r^3 \frac{P}{c^2}\right]\left[1 - \frac{2GM_r}{c^2 r}\right]^{-1}$$

This is the general relativistic analogue of the hydrostatic equilibrium equation (one restores the familiar equation when terms proportional to P are dropped). It is supplemented by the metric for a spherically symmetric distribution of matter:

$$ds^2 = e^{\nu} c^2 dt^2 - \left[1 - \frac{2GM_r}{c^2 r}\right]^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

Here ν is given by the constraint:

$$\frac{d\nu}{dr} = -\left(\frac{2}{P + \rho c^2}\right) \frac{dP}{dr}$$

Numerical integration of these equations (for $n = \frac{3}{2}$ Polytrope) leads to the M-R relation, with the minimum and maximum masses given by:

$$\boxed{\begin{aligned} M_{min} &\simeq 0.18\, M_\odot, \quad R \simeq 300\, km, \quad \rho_c \simeq 2.6 \times 10^{13}\, g\, cm^{-3} \\ M_{max} &\simeq 0.72\, M_\odot, \quad R \simeq 8.8\, km, \quad \rho_c \simeq 5.8 \times 10^{15}\, g\, cm^{-3} \end{aligned}}$$

The existence of M_{min} is due to the fact that no stable compact objects exist in the gap $10^{10}\, g\, cm^{-3} \lesssim \rho \lesssim 4 \times 10^{12}\, g\, cm^{-3}$.

The existence of M_{max} can be understood from the general relativistic stability considerations, as discussed in the case of white dwarfs.

In the case of white dwarfs, we saw that rotation can increase the maximum mass by as much as a factor of 2. The corresponding analysis in the case of neutron stars is considerably more complicated as rapidly rotating configurations and their stability criteria are not known in general relativity. If virial theorem **analysis** is repeated with the general relativistic correction as a perturbation, it can be shown that the maximum mass increases by $\sim 20\%$.

Internal Structure of Neutron Stars:

Different layers of a neutron star arise because different physical phenomena occur at different radial distances, which have different densities. The details depend on the equation of state, which is unfortunately known rather poorly (specially at very high densities). Nevertheless, some general comments can be made. To put things in perspective, for illustrative purposes, consider a $\sim 1 M_\odot$ neutron star with $R \sim 15$ km.

- Atmosphere: ~ 1 m.
- Envelope: ~ 10 m, $\rho \lesssim 10^6 \, g \, cm^{-3}$. This is a close-packed solid made of ^{56}Fe atoms. The surface magnetic field influences the structure of the atoms significantly. The solid will have high conductivity parallel to the magnetic field and zero conductivility in the perpendicular direction.
- Outer crust: ~ 0.5 km, $10^6 \, g \, cm^{-3} \lesssim \rho \lesssim 4 \times 10^{11} \, g \, cm^{-3}$. The structure is similar to that of a white dwarf. This is a solid region, with the heavy nuclei forming a Coulomb lattice embedded in

a relativistic degenerate gas of fermions. The energy of electrons can be high enough to induce inverse β-decay.

- Inner crust: ~ 2 km, $4\times10^{11}\ g\ cm^{-3} \lesssim \rho \lesssim 4\times10^{12}\ g\ cm^{-3}$. The lattice gives away to neutron rich nuclei, free neutrons, and a degenerate relativistic electron gas.

- Outer Core: ~ 9 km, $4\times10^{12}\ g\ cm^{-3} \lesssim \rho \lesssim 3\times10^{15}\ g\ cm^{-3}$. Free neutrons dominate as a result of neutron drip. Matter is in the form of a neutron liquid with a very small concentration of protons and electrons.

- Inner Core: ~ 3 km, $\rho \gtrsim 3\times10^{15}\ g\ cm^{-3}$. The existence of this phase is not generic but depends on the equation of state for bulk matter at high energies and densities. Composition and equation of state of the matter are not know well in this regime, and exotic phases like strange matter or

Pion/Kaon condensate have been contemplated.

There is one more complication that needs to be kept in mind in determining the internal structure of a neutron star. A collection of fermions can, under certain circumstances, become a superconductor (zero resistance) or a superfluid (zero viscosity). Some of the features of these phases may be of relevance in understanding neutron stars.